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On the Principles to be observed in the Construction of Mortality Tables. By WILLIAM MATTHEW MAKEHAM, Fellow of the Institute of Actuaries.

[Read before the Institute, 27th November, 1865.]

THE remarks which I have to offer for the consideration of this meeting have reference not to the deduction of the probabilities of living and dying from the facts observed, but to the mode of dealing with those probabilities, in their rough state, with the view of rendering them fit for the purpose for which they may ultimately be required.

In no observations which have hitherto been made by the exact enumeration of the living and dying at each separate year of age, have the facts observed been sufficiently numerous to render the process of adjustment unnecessary—or, at least, undesirable. Few, I think, will be disposed to contend against the propriety of adopting *some* such process; although, as might be expected, the extent to which the alteration of the original figures should be carried, is a question upon which considerable difference of opinion undoubtedly exists. The great importance of the subject will, I trust, be a sufficient apology for the following attempt to bring about some approach to an agreement upon this question.

Now I submit, with all deference, that one cause of the want

of agreement referred to may possibly arise from our not distinctly defining the precise object for which the resulting table is supposed to be required. If all we want to attain is a true statement of the rate of mortality at each age which *has* prevailed among the lives observed, it is clear that the alteration of a single figure of the actual results is inadmissible. Those figures, and those alone, give us the information we are supposed to be in search of.

But if, on the other hand, we wish to obtain, not merely the ratios of the actual number of deaths which have occurred at each age, but an idea of the law of mortality of which those figures are but the rough indications; in other words, if we wish to obtain an approximation to the ratios which would probably have resulted if the observations had been unlimited in point of number—a desideratum not only instructive in itself, but essential to a correct determination of the *probable future* experience; if this be our object, it will, I think, be admitted that a process of adjustment should be used which merely softens down the inequalities occurring at ages in immediate juxtaposition, without destroying any distinctly marked feature observable at particular periods of life, whether such feature may be supposed to belong to the normal law of mortality, to which mankind in general is subject, or whether it be considered peculiar to the particular class or community upon which the observations are based.

As an example of the kind of adjustment here referred to I may instance the observations made by the late Mr. Finlaison on the mortality of the male nominees of the Government tontines and life annuities. In observation 15 (which comprises the whole of the male lives), we find that from the age of 16 to the age of 23, the annual rate of mortality rises rapidly from about 8 to a maximum of 15 per 1,000; after which it gradually diminishes to a minimum, at age 34, of 11·7 per 1,000. This remarkable deviation from the general law of progression in the rate of mortality from age to age is not destroyed by the mode of adjustment adopted by Mr. Finlaison; and confirmed as it is by each of the separate classes of the same observations, and also by other similar but totally independent observations, there can be no question that it truly represents a distinctive feature in the mortality prevailing among the lives upon which these particular observations are based.

Such being the case, I think the conclusion is inevitable, that if our object be (as we have supposed) simply to ascertain, as nearly as possible, the true character of the law of mortality to which the community (taken in the aggregate) *has* been exposed,

and which will probably *in future* prevail among the same or a similar community; then, as before observed, a method of adjustment which (like Mr. Finlaison's) merely corrects the abrupt transitions at consecutive ages caused by the insufficiency of the data, and preserves any distinctly marked peculiarity in the results, is all that we are justified in applying. By doing more than this, the objection urged by Professor De Morgan, that the tables are thereby deprived of a portion of their value as the representations of physical *facts*, can neither be denied nor rebutted.

And here it might not unnaturally be supposed that this conclusion really involves the whole question in discussion; for, it may be asked, does not a table so adjusted form the true and proper standard for estimating the probabilities of life among the same or any similar community—the only practical object for which such tables are usually required?

To this I answer, undoubtedly it does, *provided only* (and this proviso is an important one) that nothing is known respecting the individual whose chances of life we are estimating beyond his age and the single fact that he is a member either of the particular community which has furnished the materials for the construction of the table, or of a class supposed to be subject to the same law of mortality.

For instance, we have seen that, according to the observations upon the Government male annuitants, the annual mortality at the age 23 is 15 per 1,000, while at an age 11 years greater it is 11·7 only. Now, if all we know of two particular individuals, of the respective ages 23 and 34, is that they are both included among the Government annuitants, or that they belong to the class of society from which those annuitants are principally recruited, then undoubtedly we shall rightly conclude that their relative chances of dying within a year are as 15 and 11·7; and if an Assurance Office were asked to undertake the risk upon the limited information supposed, the premium required for the younger life should exceed that required for the elder in the same proportion.

But if, on the other hand, besides the fact that the lives are Government annuitants, it is also known that both of them are at the time in sound health, of sober and temperate habits, and belong to a family free from any hereditary taint, does it *then* follow that the numbers in question will correctly denote the relative probabilities of death? That they will not truly represent the *absolute* magnitude of such probabilities is certain, and I think, for the reasons which I shall presently adduce, we shall not be far wrong

in supposing that they as little indicate their *relative* magnitude, or the ratio which the one risk bears to the other.

This brings us naturally to the third object which we may have in view in forming a table of mortality, viz., the construction of an instrument which shall serve as a measure of the value of *selected* life, that is, of the average duration of the lives of persons who, at the time when they form the subject of calculation, are known, or supposed, to be free from any exceptional cause of mortality.

In the first place, it requires but little experience to tell us that this object is of far greater general utility than either of the two which we have previously examined. It is very seldom indeed that pecuniary contracts are entered into, depending upon the contingencies of life, without *some* information being obtained respecting the state of health and the habits of the particular individual whose life is involved; and whether such information be favourable or unfavourable, it must, I think, be sufficiently obvious that a table constructed by the combination of the two classes can form but a very imperfect measure of the value of either.

It will, indeed, be readily perceived that the object which we are now considering can be perfectly attained only by an examination of the results of observations upon lives selected at each successive year of age, and the construction of a separate mortality table for each class. This subject did not escape the attention of the eminent actuaries by whom the mortality of the seventeen Assurance Offices was investigated; and I believe that in the examination which the Council of the Institute is now conducting into the present more extended experience among assured lives, the question is receiving that attention which its importance so justly demands. Mr. Higham's most able and interesting papers on this subject, read before the Institute on the 25th March, 1850, and the 31st March, 1851, are probably familiar to all present; and I think that, considering the important deductions exhibited by Mr. Higham, it must have occurred to most of us, that the observations upon the Government annuitants, valuable as they undoubtedly are, would have been rendered still more so, not only in a scientific point of view, but also as a necessary test of the adequacy of the prices charged by the Government in the grant of annuities at certain ages, if a complete investigation, conducted with the ability displayed in Mr. Higham's papers, had been made of their bearing upon this question.

Notwithstanding, however, the great interest and importance of

the subject of classification, I forbear, for two reasons, to pursue it further on this occasion. The first is, that there exist no data suitable for our purpose; for unfortunately, owing to a cause which I shall hereinafter refer to, the observations upon assured lives (important and useful as they are in other respects) do not, from their nature, admit of any correct deductions being drawn from them on this head; and secondly, I am of opinion that the complexity which would be introduced into the calculation of life contingencies, if, in each distinct class or community observed, a separate table of mortality were used for each different year of age, renders it extremely undesirable that any such practice should be adopted. The question which we have, then, to consider is, in what way we may obtain the best substitute for the perfect instrument which the circumstances of the case admit of.

Now, it appears to me that the only course open to us is, to adjust the tables which we propose to adopt for this purpose upon a somewhat broader basis than that described as applicable to the purpose previously referred to. In other words, I would first endeavour to ascertain, by an examination of the data best adapted for this object, what are the essential features of the natural or normal law of mortality, as exhibited in the effects produced by that gradual change in the vital powers of the human body which renders the chance of death so much greater to a healthy life of 70 or 80 than it is to a healthy life of 20 or 30. Having settled this question as satisfactorily as our limited means of observation will admit, we must then adjust or correct our table according to the model thus determined; and I am greatly mistaken if we shall not by these means obtain a truer standard for the value of life (having regard to the actual conditions of the case) than a more rigid adherence to the uncorrected results would have afforded us.

In order that the conclusions deduced from the investigation of the nature of the *normal* law of mortality may not be vitiated by possible errors in the collection of the data examined, I have considered it advisable to confine myself exclusively to tables formed upon exact enumerations of the numbers living and dying at each particular year of age. Whatever objections may be urged against some of the following observations in other respects, it must be admitted that in the essential point of arithmetical accuracy they are immeasurably superior to any observations derived from population returns and the public registers of deaths, however skilfully and judiciously such observations may be conducted.

The tables available for my purpose are limited (so far as my

knowledge extends) to nine, which, for reasons which will appear, I have arranged in the following order :—

(Male Life.)

1. Government Annuitants.*
2. Peerage Families.†
3. Assured Lives.‡
4. Members of Friendly Societies.§
5. Clergy of England and Wales.||

(Female Life.)

6. Assured Lives.‡
7. Friendly Societies.§
8. Government Annuitants.*
9. Peerage Families.†

I.—Observations on Male Life.—Annual Mortality per 1,000.¶

Age.	Government Annuitants.	Peerage Families.	Assured Lives.	Friendly Societies.	Clergy.	Age.
18	10·9	7·4	8·5	6·7	..	18
23	15·1	11·6	8·4	7·3	..	23
28	13·4	10·1	9·1	7·6	4·6	28
33	11·8	7·1	9·3	8·0	6·4	33
38	14·0	10·5	11·3	9·5	7·1	38
43	14·0	11·0	13·1	11·1	9·2	43
48	14·9	13·9	17·1	13·6	11·3	48
53	23·2	16·4	22·4	17·3	18·5	53
58	29·2	19·7	32·3	25·2	22·9	58
63	40·8	33·2	40·6	29·8	34·5	63
68	61·7	51·0	53·7	48·1	49·8	68
73	81·7	78·5	78·9	66·6	83·0	73
78	114·3	104·3	120·2	102·6	125·9	78

* “Report of John Finlaison, Actuary of the National Debt, on the Evidence and Elementary Facts on which the Tables of Life Annuities are founded.” Ordered by the House of Commons to be printed, 31st March, 1829.

† “On the Rate of Mortality prevailing amongst the Families of the Peerage during the 19th Century.” By Arthur Hutcheson Bailey and Archibald Day, Esqs. (*Assurance Magazine*, vol. ix.)

‡ “Tables exhibiting the Law of Mortality deduced from the Combined Experience of Seventeen Life Assurance Offices;” 1843.

§ “Report and Tables on the Sickness and Mortality among the Members of Friendly Societies.” (Alexander Glen Finlaison, Esq.) Ordered by the House of Commons to be printed, 16th August, 1853.

|| “Observations in Reference to the Duration of Life amongst the Clergy in England and Wales.” By the Rev. John Hodgson. With a Supplement by Samuel Brown, Esq. 1865.

¶ In this and the following table the annual rate of mortality at the age x is the average of the five years, $x-2$, $x-1$, x , $x+1$, and $x+2$; by which arrangement every item of the original observations has its due effect in the condensed tables here given. An exception, however, has been made in the case of the “Government Annuitants,” which are taken, without alteration, from Mr. Finlaison’s adjusted series.

II.—*Observations on Female Life.—Annual Mortality per 1,000.*

Age.	Assured Lives.	Friendly Societies.	Government Annuitants.	Peerage Families.	Age.
18	..	9.1	8.4	8.2	18
23	15.6	9.2	8.6	8.3	23
28	13.5	11.0	9.6	8.1	28
33	18.9	10.4	10.3	10.3	33
38	14.2	11.9	11.5	10.1	38
43	14.4	11.2	11.7	11.4	43
48	17.7	11.7	14.9	12.5	48
53	17.7	15.5	16.5	14.6	53
58	27.5	21.4	20.5	21.1	58
63	32.9	35.0	28.8	35.1	63
68	52.1	51.4	43.0	38.2	68
73	100.9	82.4	64.8	58.8	73
78	183.5	121.8	99.6	77.2	78

The one great and prominent characteristic common to these several observations—and, indeed, to all other observations on human mortality—is the gradual progression from age to age in the rate of mortality—the rate of increase, however, being in every instance much greater at the higher than the lower ages. This simple and obvious characteristic of the law of mortality was pointed out by Dr. Price, and indeed is consistent with the commonest notions of the nature of that law.

If, however, we examine more closely the several columns of observations upon *male* life, we find that this simple law of progression is, *in some of them*, subject to some disturbing influence, commencing apparently immediately after the age of 18 (at which our observations begin), and ceasing about the age of 38 or 40. We see, for instance, in the case of the “Government Annuitants” and the “Peerage Families” that the rate of mortality increases with considerable rapidity during the five years following the age of 18, and then gradually diminishes during the next ten years, after which it increases uninterruptedly during the remainder of the period observed. The same disturbing influence, however, is not perceptible in the other observations on male life.

If again we examine the observations upon *female* life, we find the whole of them free from this particular deviation from the general law of progression. In one of them, indeed (the assured lives), we see a diminution of the rate of mortality at age 28, but this is immediately followed by a much greater increase at age 33, after which another temporary decrease takes place. In the Friendly Societies, also, a decrease takes place twice, but this occurs at the ages 33 and 43. These fluctuations may possibly be owing

to some extent to the limited numbers upon which the observations are based ; but I am inclined to suspect that the prevalence of speculative assurances upon the lives of females—to which some Offices, I am afraid, lend themselves too incautiously—may have something to do with it. At all events, as these peculiarities are confined to the observations of this particular kind, viz., the “Assured Lives” and “Friendly Societies,” and further, as there is no uniformity in the deviations in the two observations, I think they may safely be discarded in an inquiry into the nature of the normal law of mortality.

Confining our attention, therefore, to the abnormal feature observable in two of the observations on male life immediately after the age of 18—which, in fact, is the only one existing in different and independent observations—we have now to inquire to what cause it may probably be attributable. Now, we must bear in mind that the “annuitants,” upon whose lives Mr. Finlaison’s observations are founded, were differently circumstanced from the members of Assurance and Friendly Societies, who are, for the most part, brought together, from habits of prudence and foresight, for the purpose of making a provision for the future. The “annuitants,” on the contrary, were, generally speaking (at the earlier ages), simply nominated by the person entitled to the annuity, having themselves no interest in the matter. They were, therefore, almost invariably chosen from among the families of the more affluent classes ; to some extent, because such persons would not unreasonably be supposed to enjoy a greater average duration of life, but principally because they could be more easily traced in after life. They would, in fact, belong to a class of the community very similar to that of the peerage families, in which, as we have seen, the same remarkable deviation from the law of progression is observed. Now, when we consider the mode of life, which to many young men, not in *this* particular class only, but in *all* classes of the community, at about the ages we are referring to, appears to offer such attractions—when we consider that in this particular class, many of them, from the want of any very absorbing occupation, would have more than the ordinary temptation to indulge in the excesses of such a mode of life—and further, that most of them would be possessed of ample pecuniary means for such indulgence—we need not, I think, look any further for the explanation of the mystery. But whether this conclusion be correct, or not, it is evident that the anomaly cannot be owing to any *constitutional* peculiarity in the *class*, for it is entirely absent from the

corresponding observations on female life. Nor can it arise from any peculiarity in the *sex*, for the males of other observations are free from it.

It may not, perhaps, be out of place to refer here, in passing, to the extraordinary difference which we find between the mortality of male and female life in the observations upon the Government annuitants. The authors of the very valuable observations upon the peerage families suggest that this may perhaps be owing to the probable preponderance of unmarried females in Mr. Finlaison's observations; but I think it important to point out that there are two circumstances which militate against this supposition. The first is, that although doubtless a preponderance of unmarried females would probably be found in the case of annuities purchased at the later periods of life, there is no reason to suppose that the same would be found in the lives admitted at the earlier ages, who, as previously observed, consist for the most part of the nominees in various tontines, many of whom must have been selected in childhood; yet the anomaly is the more remarkable (because unusual) at the earlier adult ages than at the more advanced periods of life. But a yet stronger reason for rejecting the hypothesis is, that by comparing the mortality of each sex with the corresponding mortality of the peerage families, we find that the great difference between the two sexes in the Government annuitants arises not from an excessive vitality in the females, but from an excessive mortality in the males—a phenomenon for which I cannot suggest any very satisfactory explanation, but the cause of which it would, I think, be very desirable to investigate.

It appears to me, then, from the considerations which I have endeavoured to explain, that, as regards the particular observations we have examined, the best data for determining the nature of the normal law of mortality—or of the variation in the rate of mortality, from age to age, which arises solely from the gradual decay of the vital powers—will be found in the males of Assurance and Friendly Societies and the clergy, and in the females of the “Government Annuitants” and “Peerage Families.”

The next point which we have to consider is the mode of adjusting the mortality table in accordance with the very simple nature of the progression found in each of the observations last referred to; in other words, of correcting the aberrations (for such, according to the views herein advanced, they must be considered) which prevail to a considerable extent in many very useful and otherwise trustworthy observations, and, indeed, to some extent

in all observations whatever. If our object be confined to this point alone, there is, I conceive, but little remaining to be done. The ablest cultivators of the science of life contingencies have devoted their powers to the elucidation of the principles upon which this should be effected; and in the works of the many eminent writers on life contingencies the student will find a vast and most instructive fund of learning and ingenuity. Most of these writers either tacitly or expressly assume the necessity of preserving an uninterrupted progression in the series representing the annual rate of mortality, the soundness of which assumption I have endeavoured to establish in the foregoing pages.

The methods of adjustment proposed by Messrs. Jellicoe and Gray (whose valuable papers on this subject have appeared in the *Assurance Magazine*) are founded upon the formula devised by Mr. Gompertz; and there can be no question that those gentlemen have satisfactorily shown that the formula in question is admirably adapted for the purpose. We occasionally hear objections expressed to the use of a mathematical formula, as an instrument of adjustment; but admitting the necessity of some process of the kind, it is difficult to see why it should not be effected by such means, provided, of course, that the result is satisfactory. Indeed, to most persons the highly philosophical principle from which Mr. Gompertz's formula is derived is, I think, sufficient to give it a decided preference over every other method.

But so far from considering the use of a mathematical formula objectionable, I look upon it as the best means of correcting the aberrations which are found in a greater or less degree in all observations, whether arising from paucity of numbers or from the existence of abnormal conditions in the data observed; a process which, as I have endeavoured to show, is essential to the proper construction of tables for computing the values of isolated contingencies.

Mr. Gompertz's theory of the law of mortality is, that the vital power, or the "power to oppose destruction," loses equal proportions in equal times; and consequently that the intensity of mortality, which is inversely proportional to this power, is represented by a series in geometrical progression. Now, if we admit the necessity of securing a gradual and progressive increase in the rate of mortality in passing from age to age, it is evident that this theory affords us a very convenient means of adjusting mortality tables; for by taking intervals of sufficient length, an increasing geometrical progression may always be secured. It has, however,

invariably been found that the ratio of progression, instead of remaining constant throughout the whole period of life, as the theory supposes, is, on the contrary, subject to a slow but continued increase with age; in consequence of which it has been found necessary to change the constants at least once, but generally twice, in the construction of a complete table of mortality. But the necessity for this change of constants may be obviated, and the required retardation in the rate of increase, at the earlier ages, may be obtained in as effectual and, I venture to think, in a more scientific manner, by supposing the intensity of mortality to be represented by a series not purely geometrical, but consisting of the sum of two terms, the one a constant quantity and the other geometrical. That is, instead of representing the intensity of mortality by an expression of the form bq^x we represent it by one of the form $c + bq^x$. By this modification of Mr. Gompertz's formula the equation for the numbers living at successive ages becomes $L_x = d.g^x.s^x$; from which it is perceived that a new constant (s) enters in the formula in a way precisely analogous to that in which the rate of interest is combined with it.

I will now briefly recapitulate the conclusions which follow from the views which I have endeavoured to enforce in the preceding pages.

First, then, it appears that there are two distinct principles to be observed in the adjustment of mortality tables—the one, which merely softens down the asperities of the original observations, without interfering with the nature of the progression at different periods of life; and the other, which imparts an uniformity of progression, consistent with the normal law of mortality, by eliminating the inherent imperfections of the data, whether arising from some of the lives being subject to exceptional causes of mortality at particular periods of life (as in the males of the Government annuitants and peerage families), or from defective means of observation, which in all probability is the case with the Carlisle table, and others similarly formed. The first of these methods is applicable when our object is merely to ascertain the nature of the law of mortality to which the lives *taken in the aggregate, or without selection*, have been and will also, probably, be in future subject; and the second, when we wish to form a standard for the determination of the probabilities of life in the case of *individuals* who are supposed to be at the time in a good state of health, and leading a life conducive to that state.

Secondly, I have endeavoured to show that certain observations

are, from their nature, much better adapted than others to afford a true representation of the normal law of mortality (by which term I mean the nature of the progression in the series denoting the intensity of mortality which is due exclusively to the decay of the vital power), and that it is precisely in those tables that a simple and uniform law of progression is most observable. These tables are—for *males*, the observations upon assured lives, the members of Friendly Societies, and the clergy; and for *females*, the Government annuitants and peerage families.

The third conclusion to which I have arrived is, that from the age of 15, or thereabouts, the normal law of mortality, of which we are in search, is characterized by an increasing progression throughout; the rate of increase, however, being at first very slow, and gradually gaining in rapidity with increased age. It is this characteristic which renders the formula before described (consisting of a constant combined with an increasing geometrical series) singularly well adapted to represent the law in question from adolescence to extreme old age—a satisfactory proof of which assertion I hope to give on a future occasion, when I propose also to examine the results of an extension of the formula to all periods of life.

I wish it to be distinctly understood, nevertheless, that I do not by any means contend that a table, such, for instance, as the males of the Government annuitants or the peerage families, even when corrected by means of the formula which I propose, will form a *perfect* instrument for the purpose of estimating the contingencies of individual selected life. On the contrary, I have stated that it is only by observations on those lives classified according to the age of selection that such a desideratum can be obtained. I only argue that, so corrected, the table will be a better standard than when adjusted upon a narrower or more restricted basis.

Indeed, I incline to the opinion that the selection of the table of mortality best adapted for estimating the value of a particular life must ever be, to a considerable extent, arbitrary, or a matter of opinion. In calculations involving the interest of money we have a few broadly distinguished rates upon which we base our calculations, the particular rate selected being a subject of previous agreement between the parties to the contract. We never think it necessary to make the subdivisions of the rate of interest less than one quarter per cent., and scarcely ever indeed go even to that extent—and this because we know that it is impossible to determine the real future value of money to such a degree of accuracy.

Now, it appears to me that we are situated very similarly with regard to the mortality table; and that for practical purposes what we require is, some three or four broadly marked tables to serve as the standards of value, founded upon the best observations, but corrected as nearly as possible according to what appears to be the natural or normal law of mortality. Let us consider for a moment the probable effect which the different positions of individuals, even in the same class of society (not to mention hundreds of other equally important considerations), must have upon the chances of life. Yet a calculator will proceed to make his valuation without inquiring whether the life proposed is a person who is free from any harassing cares, and has nothing to do but to study his health and comfort, or whether he is burthened with the responsibilities attaching to an onerous and important office; nor could he do much with the information if he obtained it. To stipulate then, as an essential condition, for a calculation in strict accordance with the data supplied by any set of observations, may bear an *ad captandum* appearance of accuracy; but, from what I have stated, such an appearance must be entirely illusory, and such a stipulation is, I think, scarcely consistent with a philosophical or scientific view of the subject.

I must, however, guard myself against the risk of being supposed to underrate the value of tables showing the mortality prevailing in specific classes of the community—such, for instance, as the “Peerage Families” and the “Clergy.” Such tables I consider of the greatest possible importance, as indicating the result upon the value of life of the peculiar conditions which affect different classes of society; and we are under the deepest obligation to the ingenious authors of these valuable tables. Neither can there be the slightest objection to the use of such tables in computing the pecuniary values of contingencies upon lives belonging to these respective classes; and, indeed, when our information respecting the life is restricted to the condition which forms the basis of the table, such table will indicate precisely the true value of the given contingency. But as it will seldom happen that our information *is* so restricted—as we shall generally know something about the individual which will induce us to think that the true value of the life must differ from the value indicated by the table quite as much as the best tables are found to differ from one another—it is, I think, in the former rather than in the latter capacity, as guides rather than as instruments of calculation, that the chief value of these tables will be found to consist.

It is, perhaps, from some such view of the matter as that which I am endeavouring to explain that we must account for the hold which the Carlisle Table still retains as a measure of the value of life contingencies in this country. With all its faults of adjustment it has been found to agree *in the main* with observations founded upon much better principles and upon a more extended basis; and it is felt that having been so generally adopted as a standard it would be but an idle *affectation* of refinement to discard it and the many useful tables which have been formed upon it, for tables of the latter description. It is for this reason that, in the construction of the table which I now submit for the consideration of the members, I have adopted the Carlisle table as a basis, and I now proceed to show that the alterations which I have introduced by the process of readjustment are merely corrections of the defects of that celebrated table.

I may here observe that Mr. Milne's adjustment of the Carlisle table is, in my opinion, a striking instance of the misapplication of the first of the two methods described in the foregoing pages. The table was intended, not simply as a representation of the law of mortality which had prevailed amongst the particular community observed, but for the far more important object of furnishing a better standard for the valuation of life contingencies than any then in use. Instead, therefore, of adopting the method of adjustment first described, whereby the final table exhibits all the peculiar and abnormal features of those particular observations—and which, as I shall show, are in all probability due to errors in the mode of collecting the data—Mr. Milne should have kept to the broad and simple features common to *all* observations, as described by Dr. Price, and as exemplified in all the tables then in use. His table would then have assumed somewhat of the form of the readjusted table which I have constructed by the use of the formula herein described.

I cannot help thinking that Mr. Milne must himself have been partly sensible of the distinction which I have endeavoured to draw between these two modes of adjustment; for while he has been careful to preserve in his mortality table all the irregularities which his data exhibited, yet I cannot find that he computed his tables of premiums for “term” assurances (in which alone these irregularities would be visible) upon the same principle. In the tables of premiums published at the end of the second volume of Mr. David Jones' work, I find that the rates of the Sun Office for assurances for a single year are in increasing progression through-

out. Now Mr. Milne could hardly have been so inconsistent as to hold that it is necessary to adhere strictly to the data in the case of "whole life" premiums—where the results of adjustment are comparatively insignificant—and to abandon them in the case of "term" premiums, in which alone the process of adjustment makes any material difference. It is more reasonable to suppose that he felt the inapplicability of his method of adjustment to a table intended for the calculation of assurance premiums.

But a stronger argument in favour of a readjustment of the Carlisle table will, I think, be found in the following somewhat curious circumstance connected with the construction of that celebrated table, which I have never yet seen noticed. From Mr. Milne's observations on the method of constructing mortality tables, in his first volume, and his explanation respecting the application of the method to the Carlisle table, in the second, we may, I think, conclude that he believed Dr. Heysham's observations upon the inhabitants of Carlisle in the years 1780 and 1787 were founded in each case upon enumerations of the numbers living in the several stated periods, and not of the total number of the population only. With regard to the enumeration of 1780, which was made by Dr. Heysham himself, that gentleman says (Milne, vol. ii., p. 746), "When I made the survey of Carlisle, in the beginning of the year 1780, there were between [the ages of] 10 and 15, 715; and between 15 and 20 years of age, 675," &c. From this we may infer that the survey of 1780 comprised a complete enumeration of the ages as well as the number of the inhabitants; but is it equally clear that this was so with the survey of 1787?

The latter enumeration was made in pursuance of an order from the Court of Quarter Sessions to the different constables in the county of Cumberland, "to make an actual survey of all the inhabitants of the county." This is pretty nearly all the information given on the subject in Mr. Milne's abridgment of Dr. Heysham's pamphlet; and nowhere is it therein stated whether the enumeration took account of the ages of the inhabitants. The original pamphlet of Dr. Heysham I have not the means of consulting.

The first three columns of the following table contain the data upon which Mr. Milne determined the numbers exposed to the risk of mortality during the eight years comprised in Dr. Heysham's observations.

III.—Population of Carlisle.

Between the Ages of	NUMBER LIVING IN		
	Jan., 1780.	Dec., 1787.	
(1)	(2)	(3)	(4)
0 and 5	1,029	1,164	1,163
5 „ 10	908	1,026	1,026
10 „ 15	715	808	808
15 „ 20	675	763	763
20 „ 30	1,328	1,501	1,501
30 „ 40	877	991	991
40 „ 50	858	970	970
50 „ 60	588	665	665
60 „ 70	438	494	495
70 „ 80	191	216	216
80 „ 90	58	66	66
90 „ 100	10	11	11
100 „ 105	2	2	2
Total	7,677	8,677	8,677

The first column contains the several intervals of age; the second, the numbers living in each interval in January, 1780; and the third, the numbers living in December, 1787; the fourth and last column, which I have added myself, is derived from the second, by increasing each item in the exact proportion in which the total population had increased during the interval, that is, I have multiplied each item in column 2 by $\frac{8677}{7677}$.

By a comparison of this last column with the one immediately preceding it, I hold it to be a fair assumption (until positive evidence of the contrary be produced) that the latter was obtained, not by an enumeration of the inhabitants according to age, but by the same method as that by which I have constructed the fourth column. The two instances in which the results differ (in each case by unity only) are, perhaps, just sufficient to show that the proportions were not determined by an actuary.

Dr. Heysham estimates that of the increase of 1,000 inhabitants (in a population of 7,677), which took place during the period observed, more than one-half (viz., 511) arose from an excess of new settlers over emigrants. The actual disturbance in the population is very inadequately represented by the number in question, which is the *difference* only between the numbers of immigrants and emigrants; the actual number of the former must have been

much greater. Now, these opposing forces, it is assumed in the construction of Mr. Milne's table, were so nicely regulated as to distribute the excess in the exact proportion to the ages of the existing population. May we not, therefore, fairly suspect that the irregularities of the Carlisle table may be in some measure attributable to the perverseness of these two bodies—the new settlers and the emigrants—in not regulating their movements in accordance with so convenient an hypothesis?

Proceeding now with the explanation of the modifications which I have introduced by my process of readjustment, I have to beg attention to the following table, which exhibits in one view the original and readjusted Carlisle table, together with the results of the principal observations previously examined; the latter, for convenience of comparison being combined as follows:—Males of the Government annuitants with those of the peerage families, males of the Assurance Office with those of the Friendly Societies, and females of the Government annuitants with those of the peerage families.

IV.—*Annual Mortality per 1,000.*

Age.	Government Annuitants and Peerage. (Males.)	Carlisle.	Ditto, Readjusted.	Friendly Societies and Assured Lives. (Males.)	Government Annuitants and Peerage. (Females.)	Age.
18	9.2	7.0	7.6	7.6	8.3	18
23	13.4	7.0	8.0	7.9	8.5	23
28	11.8	8.7	8.5	8.4	8.9	28
33	9.5	10.1	9.3	8.7	10.3	33
38	12.3	11.2	10.4	10.4	10.8	38
43	12.5	14.6	12.3	12.1	11.6	43
48	14.4	13.9	15.0	15.4	13.7	48
53	19.8	16.1	19.3	19.9	15.6	53
58	24.5	24.2	25.7	28.8	20.8	58
63	37.0	38.3	35.6	35.2	32.0	63
68	56.4	46.5	50.4	50.9	40.6	68
73	80.1	78.1	72.7	72.8	61.8	73
78	109.3	108.8	105.8	111.4	88.4	78

Upon examining the above table we remark that the deviations from the normal law of progression observable in the Carlisle data are mostly of an entirely opposite character to those of the Government annuitants and peerage families (males). Thus, instead of an undue increase from 18 to 23, followed by a decrease until 33, we find, on the contrary, that the first abnormal increase commences at 28 and continues until 33. Again, in the Government annuitants and peerage families (males) a rapid increase takes place from 33 to 38, followed by a suspended mortality in the

next five years. In the Carlisle table, on the contrary, the increase is normal from 33 to 38, but an undue increase occurs from 38 to 43, followed by a decrease in the next five years. These inconsistencies in Mr. Milne's table I have endeavoured to show are, in all probability, due to defective means of observation; and if so, it follows that the very great pains which that able author appears to have taken in the adjustment of his table were not only unnecessary, but have rendered it far less correct (even when considered merely as a representation of *physical facts*) than if he had adopted the simpler and less laborious processes used by his predecessors.

It will also be seen that the readjusted Carlisle table follows very closely the mean of the "Assured Lives" and "Friendly Societies." The only ages at which there is any material difference are 33, 58, and 78; and it will be seen that at these ages the readjusted Carlisle table follows more nearly than the "mean" table of "Assured Lives" and "Friendly Societies" the general run of the *other* observations; from which it is not unreasonable to infer that the "mean" table in question is to some extent abnormal at these particular ages. Thus, at age 33, where the "mean" is *below* the "readjusted" table, it is also below every other observation; at 58, where it is considerably *above*, it is still more in excess of every other; and again, at 78, where it is also *above*, it will again be found to be higher than all the others.

It may not be out of place to refer here to the great superiority in point of vitality of the males of "Friendly Societies" over those of the "Assured Lives." The explanatory remarks prefixed to the observations on "Assured Lives" are altogether silent on the subject of the mode in which lives taken at special rates of premium—whether on account of deteriorated health or foreign residence—were treated; from which, I presume, we must infer that no distinction was made between these and other cases. If so, this would doubtless account, to some extent, for the superiority in question. Further, there can, I think, be little doubt that owing to the withdrawal of many of the best lives by the surrender and discontinuance of policies, the mortality among assured lives is higher than it would have been if the whole of the members had remained on the books, while in the Friendly Societies the discontinuance of membership is supposed to have a contrary effect. Nor is this seeming contradiction (if it exists) impossible to explain. The member of an Assurance Society, if his health fails him, will exhaust every means to keep up his policy; and should he find it impossible to accomplish this, he will have little difficulty in finding

a purchaser, among the many speculators in such property, upon better terms than the rules of the Office will afford him. The member of a Friendly Society, on the other hand, if he becomes unable to continue his payments, as *his* policy is not a marketable commodity, has no alternative but to forfeit his claim upon the Society. Again, it is no unusual thing to find, when a member of an Assurance Society becomes invalided, destitute, or of intemperate habits, the friends of the wife will keep up the payments, if only to prevent the family becoming a burthen upon them. Such is not, I apprehend, generally speaking, the case with the member of a Friendly Society. His friends, if they are as willing, are less able to help him in his necessity. Besides, a considerable portion of his subscription, I believe, is expended in the enjoyments of convivial meetings; and thus his membership partakes of the nature of a luxury. His friends therefore, even if they were as able, would on this account be less disposed to tax themselves in order to maintain it.

As an illustration of the closeness with which the formula proposed, while, as I have endeavoured to show, it improves the series exhibiting the rate of mortality from age to age, at the same time preserves the *general effect* of the observations dealt with, I submit the following table of the mean duration of life by the Carlisle table and also by the readjusted table. It will thus be seen that the greatest difference between the two (under the age of 88, beyond which I consider any comparison useless) is $\cdot 35$, occurring at ages 47 and 48. Now, if we refer to Mr. Gray's very able paper, in vol. vii. of the *Assurance Magazine*, wherein that gentleman gives a table constructed by the application of Mr. Gompertz's formula to the same table, we find the greatest divergence to be $\cdot 28$, which occurs at the ages 66 and 67. But the interval taken by Mr. Gray is 10 years only, which he is enabled to do by changing the constants *twice* during the process. The interval I am compelled to adopt, owing to the use of a formula with four *unchanged* constants, is 25 years, or two and a half times as great as Mr. Gray's; while the greatest divergence in my table exceeds that of Mr. Gray's by only one-fourth. And looking at it merely as a question of the comparative closeness of the agreement, I think I may say that the "very awkward break" which Mr. Gray points out as occurring in his table at age 50, is got rid of in my table with no very great sacrifice of approximate accuracy. In Mr. Jellicoe's excellent adjustment of the Eagle Experience, the greatest difference between the unadjusted and the adjusted results

is .76, which occurs at age 22. In this case, however, it must be observed that there was no preliminary adjustment of the rough data previous to the application of the formula used by Mr. Jellicoe, as in the case of the Carlisle table. I need scarcely add, that my object in making these comparisons is to show that, considering my formula simply as an instrument for *adjusting* tables of mortality, the alterations of the original data are not significantly greater than those introduced by the methods adopted by the most eminent authorities.

V.—*Mean Duration of Life.*

Age.	Carlisle Table.	Same Readjusted.	Age.	Carlisle Table.	Same Readjusted.	Age.	Carlisle Table.	Same Readjusted.
15	45.00	44.88	46	23.82	23.50	77	6.40	6.41
16	44.27	44.22	47	23.17	22.82	78	6.12	6.05
17	43.57	43.55	48	22.50	22.15	79	5.80	5.71
18	42.87	42.88	49	21.81	21.48	80	5.51	5.38
19	42.17	42.21	50	21.11	20.81	81	5.21	5.07
20	41.46	41.53	51	20.39	20.16	82	4.93	4.77
21	40.75	40.85	52	19.68	19.50	83	4.65	4.48
22	40.04	40.17	53	18.97	18.86	84	4.39	4.21
23	39.31	39.48	54	18.28	18.22	85	4.12	3.95
24	38.59	38.80	55	17.58	17.59	86	3.90	3.70
25	37.86	38.11	56	16.89	16.96	87	3.71	3.47
26	37.14	37.42	57	16.21	16.35	88	3.59	3.24
27	36.41	36.72	58	15.55	15.74	89	3.47	3.03
28	35.69	36.03	59	14.92	15.14	90	3.28	2.83
29	35.00	35.33	60	14.34	14.56	91	3.26	2.65
30	34.34	34.64	61	13.82	13.98	92	3.37	2.47
31	33.68	33.94	62	13.31	13.41	93	3.48	2.30
32	33.03	33.24	63	12.81	12.86	94	3.53	2.15
33	32.36	32.54	64	12.30	12.31	95	3.53	2.00
34	31.68	31.84	65	11.79	11.78	96	3.46	1.87
35	31.00	31.14	66	11.27	11.26	97	3.28	1.74
36	30.32	30.44	67	10.75	10.75	98	3.07	1.62
37	29.64	29.73	68	10.23	10.25	99	2.77	1.51
38	28.96	29.03	69	9.70	9.77	100	2.28	1.41
39	28.28	28.34	70	9.18	9.30	101	1.79	1.31
40	27.61	27.64	71	8.65	8.85	102	1.30	1.22
41	26.97	26.94	72	8.16	8.41	103	.83	1.14
42	26.34	26.25	73	7.72	7.98	104	.50	1.07
43	25.71	25.56	74	7.33	7.57	105	..	1.00
44	25.09	24.87	75	7.01	7.17	106	..	.94
45	24.46	24.18	76	6.69	6.78	107	..	.88

Reverting now to Mr. Higham's paper on the effect of selection among assured lives, while I admire the skill displayed in the treatment of his data I must express my decided dissent from some of the conclusions which he draws from them; such, for instance, as the justification which he imagines he finds in his results for the use of the Northampton 3 per cent. table in the calculation of premiums for assurance. Mr. Higham, it appears to me, has not

considered in its proper light an important element in the observations on assured lives, the existence of which renders such observations unsuitable for the determination of rates of premium, whether taken in the aggregate or in separate classes according to the age at admission. To render them suitable for this purpose it would be necessary, after classifying the lives in the way referred to, to trace each life during the remainder of the period observed. But it is a well known fact that many lives are prematurely withdrawn from observation by the discontinuance or surrender of the policy, and such withdrawals necessarily consist of *select* lives—that is, of course, select at the time of withdrawal—for, as previously observed, the deteriorated and doubtful lives do not withdraw.* This constant draining of the better class of lives must necessarily have the effect of materially increasing the rate of mortality among the lives which remain upon the books; a result which, indeed, is sufficiently evident from the following comparison between the mean duration of life according to Mr. Higham's "Class Mortality," where the lives are *selected* at the given age, and also according to Mr. Milne's Carlisle table.

VI.—*Mean Duration of Life.*

Age.	Assured Lives.	Carlisle.
25	36·50	37·86
30	33·48	34·34
35	30·38	31·00
40	27·19	27·61
45	23·77	24·46
50	20·49	21·11
55	17·35	17·58
60	14·54	14·34
65	11·83	11·79
70	9·47	9·18
75	7·62	7·01

Here we see that at the age 60 and *upwards* the "class" mortality is in every instance more favourable than the Carlisle table, while going *backwards* towards the younger ages we find the contrary result—the vitality of the "class" observations being below the Carlisle, and the difference increases as we approach the younger ages. Now, what ought we to infer from this fact? Evidently, *not* that the lives admitted at the age 25, for instance,

* The observations on the "Government Annuitants," if the elementary facts were accessible to the public, would furnish the means of testing the effect of selection without the drawback here adverted to.

were not of that select description at the time of admission which we suppose them to be, but that their aggregate vitality has been reduced by the constant drain of the best lives, which has been going on since the time of admission by the discontinuance and surrender of policies.

This, indeed, is the conclusion arrived at by Mr. Higham, and so far, therefore, I perfectly agree with him. But he goes on to argue that we ought therefore to calculate our rates of premium accordingly. That is, in order to provide for the loss occasioned by the probable withdrawal of *some* of the lives, an additional tax should be levied, from the commencement, upon the *whole*. This, of course, is *one* way of meeting the difficulty, but it is not, in my opinion, the proper way.

The only safe and equitable plan in matters of this kind is, to follow the simple rule which Mr. Gompertz so strongly insists upon in his paper, read before the Royal Society in June, 1820, viz., that we must first make our estimate according to the real facts of the case, and upon the most *accurate* elements procurable, both as regards the rate of interest and the rate of mortality, and then make a margin for security and contingencies, in whichever direction the same may be necessary. Thus, in calculating the rate of premium, we must treat the life (as it really is) as a select life, *adding* the margin for contingencies; and in valuing the policy for surrender, we must also proceed upon the assumption of the life being *then* a select one, *deducting* in this case the margin for contingencies. By this arrangement the retiring policyholder pays (as he ought to do) for the disturbance in the average quality of the remaining lives which he occasions. If Mr. Gompertz's sound and rational maxim had been earlier understood and acted upon, we should never have heard of the preposterous terms which some Offices even yet allow as an inducement for their best lives to leave them, thereby inflicting an injury, not upon themselves only, but upon the family of the surrendering policyholder, who might otherwise be induced to make an effort to keep up his payments.

In reference to this subject, however, it is important to bear in mind, that whichever mode we may adopt to protect the Office against the *loss* occasioned by the deterioration of its lives by the surrender of policies, yet the fact of the deterioration remains, and must therefore be taken into account in making an estimate of the outstanding liabilities of an Office. Great judgment and discrimination are therefore evidently required in determining the table of mortality to be used in a distribution of profits, the more so as a very great

difference in the amount of the estimated surplus will result from the use of different tables for this purpose.

In conclusion, I have merely to add, that out of different modifications of Mr. Gompertz's formula which I have tried for the purpose of adjustment, I have been determined in my selection of the one herein proposed chiefly by the fact of its affording a very important aid in the construction of tables for annuities and assurances involving two or more lives. This is effected by means of a property analogous to that of "uniform seniority," which Professor De Morgan has shown to belong to the formula of Mr. Gompertz, and which latter property is described as follows:—If x and $x+h$ represent the ages of two given lives, then the annuity on the joint existence of those lives will be precisely equal to an annuity on a single life whose age is $x + \frac{\log(1+q^h)}{\log q}$. In the modification of

Mr. Gompertz's formula which I have adopted the same law holds good; but instead of being equal to an annuity on a single life, the required joint life annuity is equal to an annuity on two equal joint lives whose common age is $x + \frac{\log \frac{1+q^h}{2}}{\log q}$. And generally the

annuity on any number (n) of joint lives, aged respectively $x, x+h, x+k, x+l \dots$, will be equal to an annuity on the same number of equal joint lives whose common age is $x + \frac{\log \frac{1+q^h+q^k+q^l+\dots}{n}}{\log q}$.

It will not, however, be necessary (in practice) to calculate the value of this expression, as, by means of the equation $q^x + q^{x+h} + q^{x+k} + q^{x+l} + \dots = nq^r$ (where r is the equivalent common age), and tables of annuities for every value of nq^r , we have a simpler method of obtaining the value of the required annuity. The full elucidation of this principle, and the advantages afforded by it, will form the subject of another paper, which I hope to have the honour, on some future occasion, of reading before the members of this Institute.

Having thus endeavoured to establish the principles which it appears to me should always be kept in view in constructing tables of mortality, I propose, in the next division of my subject, to exemplify those principles by practical applications of a method of graduation to some of the preceding tables. The third and concluding portion will comprise an explanation of the method of constructing tables for solving the various problems in life contingencies upon the basis of the property mentioned in the preceding paragraph.